Second Fundamental Theorem of Calculus

Let $f(x)$ be a real continuous function of real variable x (see Figure 1) on interval I and a any point in I.

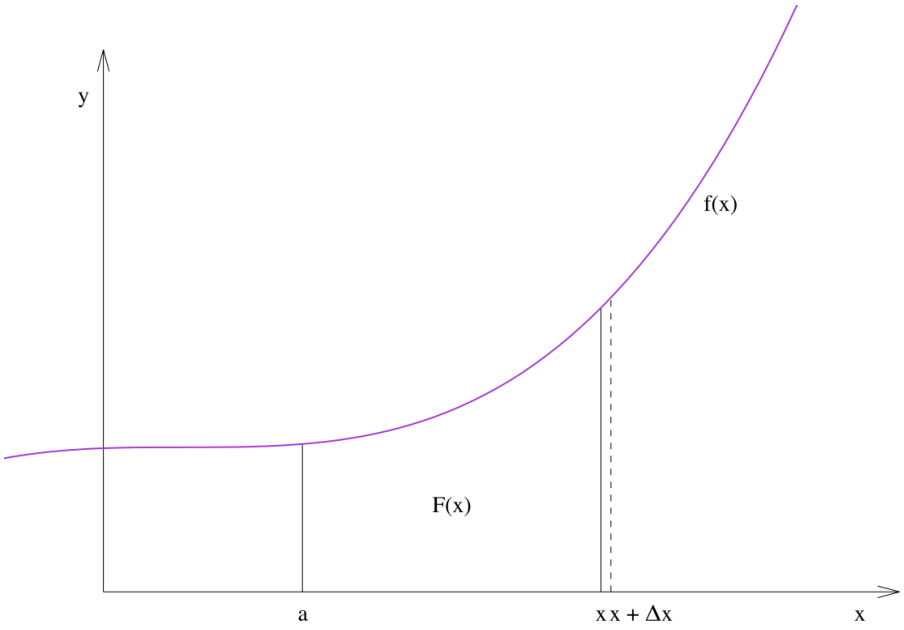

Figure 1: The illustration of the Second Fundamental Theorem of Calculus

We denote by $F(x)$ the area under the curve $f(x)$ and between the segments $(a, f(a))$, (a, x), and $(x, f(x))$. We have then

$$F'(x) = \frac{dF(x)}{dx} = \lim_{\Delta x \to 0} \frac{F(x + \Delta x) - F(x)}{\Delta x} \tag{1}$$

On the other hand

$$\lim_{\Delta x \to 0} F(x + \Delta x) - F(x) = \lim_{\Delta x \to 0} \Delta x \, f(x) \tag{2}$$

and when we divide the above equation by Δx we receive

$$\lim_{\Delta x \to 0} \frac{F(x + \Delta x) - F(x)}{\Delta x} = f(x) \tag{3}$$

and

$$F'(x) = f(x) \tag{4}$$

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