Note on square impulse Fourier transform

Let us consider a function $f(t)$

$$f(t) = \begin{cases} 1, & \text{if } |t| < 1 \\ 0, & \text{if } |t| \geq 1 \end{cases} \tag{1}$$

Then let us compute the Fourier transform

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t}\, dt = \int_{-1}^{1} e^{-i\omega t}\, dt = \frac{e^{-i\omega} - e^{i\omega}}{-i\omega} = \frac{2\sin(\omega)}{\omega} \tag{2}$$

We invert the fourier transform and receive

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{i\omega t}\, d\omega \tag{3}$$

When $t = 0$

$$\begin{aligned} f(t=0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{i\omega 0}\, d\omega &= \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)\, d\omega \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{2\sin(\omega)}{\omega}\, d\omega = 1 \end{aligned} \tag{4}$$

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