Note on product of functions derivative $(f(x)g(x))'$

Let us consider the product $f(x)g(x)$ of two functions $f(x)$ and $g(x)$ and try to compute the derivative $(f(x)g(x))'$ of their product.

Using the definition of the derivative we write

$$(f(x)g(x))' = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)g(x+\Delta x) - f(x)g(x)}{\Delta x} \tag{1}$$

$$\begin{aligned}(f(x)g(x))' = &\lim_{\Delta x \to 0} \frac{f(x+\Delta x)g(x+\Delta x) - f(x)g(x+\Delta x)}{\Delta x} \\ &+ \lim_{\Delta x \to 0} \frac{f(x)g(x+\Delta x) - f(x)g(x)}{\Delta x} \\ = &\lim_{\Delta x \to 0} \frac{(f(x+\Delta x) - f(x))g(x+\Delta x)}{\Delta x} + \lim_{\Delta x \to 0} \frac{f(x)(g(x+\Delta x) - g(x))}{\Delta x}\end{aligned} \tag{2}$$

and we conclude that

$$(f(x)g(x))' = f'(x)g(x) + f(x)g'(x) \tag{3}$$

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