Number of subsets having k elements chosen from a set of N elements

We can compute the number of sets of k elements chosen from a set of N elements as follows. We chose the first of k elements from N elements in N ways, we chose the second of k elements from N elements in $N-1$ ways, and so on, till we chose the k-th element in $N-(k-1) = N-k+1$ ways.

$N*(N-1)*(N-2)*\ldots*(N-k+1)$ is the number of ordered sequences of k elements chosen from N elements. If we do not take into account the particular sequences and take into account just the subset of k elements from the set of N elements, we need to divide $N*(N-1)*(N-2)*\ldots*(N-k+1)$ by the number of all sequences of k elements, i.e. we need to divide $N*(N-1)*(N-2)*\ldots*(N-k+1)$ by $k!$. Then the number of all subsets having k elements chosen from a set having N elements is equal to

$$\frac{N*(N-1)*(N-2)*\ldots*(N-k+1)}{k!} = \frac{N!}{(N-k)!k!} = \binom{n}{k} \qquad (1)$$

The above equation has the definition of the Newton symbol $\binom{n}{k}$. We read it N choose k. The number of sequences consisting of a set of k different elements is equal to $k*(k-1)*(k-2)*\ldots*1 = k!$ We chose the first element out of k elements in k ways, the second element out of $k-1$ elements in $k-1$ ways, and so on and the k-th element out of one element just in one way.

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