Number of subsets of a set of N elements

The number of sets of k elements from a set of N elements is

$$\binom{n}{k} \tag{1}$$

The number of all possible sets from a set consisting of N elements is

$$\sum_{k=0}^{N} \binom{n}{k} \tag{2}$$

We have the binomial theorem

$$(a+b)^N = \sum_{k=0}^{N} \binom{n}{k} a^k b^{n-k} \tag{3}$$

We notice that if $a = 1$ and $b = 1$

$$(1+1)^N = \sum_{k=0}^{N} \binom{n}{k} 1^k 1^{n-k} = \sum_{k=0}^{N} \binom{n}{k} \tag{4}$$

what gives

$$\sum_{k=0}^{N} \binom{n}{k} = 2^N \tag{5}$$

Let us have a set A of three elements $A = \{a, b, c\}$. The subsets of A are empty set $\{\emptyset\}$, sets $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$, $\{b, c\}$, and $\{a, b, c\}$. The set A has three elements and the number of all subsets of the set A is equal to $2^3 = 8$.

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