Locus example: circle

Locus of point $P(x, y)$ satisfying given condition

$$F(x, y) = f(x, y) \tag{1}$$

we call the set of all points coordinates of which are fulfilling the equation (1). Equation (1) we call the *equation of locus.*

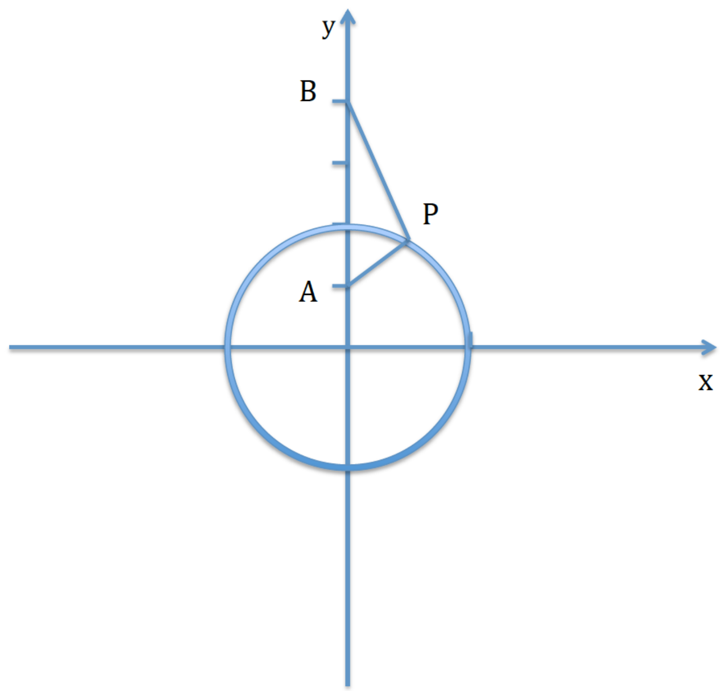

Figure 1: The distance of point $P(x, y)$ from point $A(0, 1)$ is two times smaller than the distance of point $P(x, y)$ from point $B(0, 4)$.

Let us find locus of point $P(x, y)$, the distance of which to the point $A(0, 1)$ is two times smaller than its distance to the point $B(0, 4)$ (Figure 1).

For the moving point $P(x, y)$ the locus of which we try to find there occurs relation

$$2|\overline{AP}| = |\overline{BP}| \tag{2}$$

Raising to the power of 2 we receive

$$4|\overline{AP}|^2 = |\overline{BP}|^2 \tag{3}$$

or

$$4[x^2 + (y-1)^2] = x^2 + (4-y)^2 \tag{4}$$

$$\begin{aligned} 4x^2 + [2(y-1)]^2 &= x^2 + (4-y)^2 \\ 3x^2 + [2(y-1)]^2 - (4-y)^2 &= 0 \end{aligned} \tag{5}$$

Now we use the formula $a^2 - b^2 = (a+b)(a-b)$ where $a = 2(y-1)$ and $b = (4-y)$ what gives

$$\begin{aligned} 3x^2 + [2(y-1) + 4 - y][2(y-1) - (4-y)] &= 0 \\ 3x^2 + (y+2)(3y-6) &= 0 \\ 3x^2 + 3y^2 - 6y + 6y - 12 &= 0 \\ 3x^2 + 3y^2 - 12 &= 0 \end{aligned} \tag{6}$$

and we receive

$$x^2 + y^2 = 4 \tag{7}$$

what is equation of a circle. The coordinates of the point $P(x, y)$ are satisfying the equation (7). We have then found the locus of point $P(x, y)$ fulfilling the condition (2). The equation of locus of point $P(x, y)$ is the equation of circle of the radius equal to 2 and the center at the point (0, 0).

References

[1] Swietoslaw Romanowski and Wlodzimierz Wrona (1967) *Matematyka wyzsza dla studiow technicznych* Warszawa, Panstwowe Wydawnictwo Naukowe

Pawel Jan Piskorz (paweljs@gmail.com)