

Note on harmonic series

The harmonic series is

$$\sum_{i=1}^{\infty} \frac{1}{i} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} + \frac{1}{10} + \dots \quad (1)$$

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} + \dots \quad (2)$$

$$> 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{16} + \dots \quad (3)$$

The terms in the series (3) can be added as below

$$1 + \frac{1}{2} + \left(\frac{1}{4} + \frac{1}{4}\right) + \left(\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}\right) + \left(\frac{1}{16} + \dots + \frac{1}{16}\right) + \dots \quad (4)$$

In the series (4) there is one term with $\frac{1}{2}$, two terms with $\frac{1}{4}$, four terms with $\frac{1}{8}$, eight terms with $\frac{1}{16}$, and so on, what gives

$$1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \dots = \infty \quad (5)$$

The harmonic series is greater than the series (5) which sums to the infinity, what means that the harmonic series also sums to the infinity or in other words it is divergent.

References

- [1] [https://en.wikipedia.org/wiki/Harmonic_series_\(mathematics\)](https://en.wikipedia.org/wiki/Harmonic_series_(mathematics))

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