Euler gamma function of 1/2

The gamma function is defined as

$$\Gamma(n) = \int_0^\infty x^{n-1} e^{-x}\, dx \tag{1}$$

$$\Gamma\left(\frac{1}{2}\right) = \int_0^\infty x^{\frac{1}{2}-1} e^{-x}\, dx = \int_0^\infty x^{-\frac{1}{2}} e^{-x}\, dx = \left| \begin{array}{rcl} x^{\frac{1}{2}} & = & u \\ \frac{1}{2} x^{-\frac{1}{2}}\, dx & = & du \\ x^{-\frac{1}{2}}\, dx & = & 2du \\ x & = & u^2 \end{array} \right| \tag{2}$$

$$= 2\int_0^\infty e^{-u^2}\, du = 2 \cdot \frac{1}{2}\sqrt{\pi} = \sqrt{\pi}$$

We have then the result

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi} \tag{3}$$

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