Euler gamma function alternative expression

The gamma function is defined as

$$\Gamma(n) = \int_0^\infty x^{n-1} e^{-x}\, dx \tag{1}$$

We can write then

$$\Gamma(n) = \int_0^\infty x^{n-1} e^{-x}\, dx = \left| \begin{array}{rcl} e^{-x} & = & v \\ x = 0 & & v = 1 \\ x = \infty & & v = 0 \\ \ln v & = & -x \\ x & = & -\ln v \\ \frac{dx}{dv} & = & -\frac{1}{v} \\ dx & = & -\frac{1}{v}\, dv \end{array} \right| \tag{2}$$

$$= \int_1^0 (-\ln v)^{n-1} v \Big(-\frac{1}{v} \Big)\, dv = \int_0^1 (-\ln v)^{n-1}\, dv$$

The alternative expression for Euler gamma function is

$$\Gamma(n) = \int_0^1 (-\ln v)^{n-1}\, dv \tag{3}$$

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