Note on eigenvalues and solving $Ax(t) = x'(t)$

$$\begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1' \\ x_2' \end{bmatrix} \tag{1}$$

We need to calculate the eigenvalues of the matrix

$$\begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} \tag{2}$$

In order to do that we write

$$\begin{vmatrix} 1-\lambda & 1 \\ 4 & 1-\lambda \end{vmatrix} = (1-\lambda)^2 - 4 = (\lambda+1)(\lambda-3) = 0 \tag{3}$$

and we receive the eigenvalues $\lambda_1 = -1$ and $\lambda_2 = 3$.

Let us take the equation

$$\begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} e^{-t} = \left(\begin{bmatrix} a \\ b \end{bmatrix} e^{-t} \right)' = \begin{bmatrix} -a \\ -b \end{bmatrix} e^{-t} \tag{4}$$

We receive two equations for a and b

$$a + b = -a \tag{5}$$

$$4a + b = -b \tag{6}$$

and the relation $-b = 2a$, so if $a = 1$ then $b = -2$

For the eigenvalue $\lambda_1 = -1$ we have

$$\begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{-t} = (-1) \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{-t} \tag{7}$$

Also for the eigenvalue $\lambda_2 = 3$ we receive

$$\begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{3t} = 3 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{3t} \tag{8}$$

The equtions equation (7) and equation (8) are equations corresponding to eigenvalues -1 and 3

$$Ax = \lambda x \tag{9}$$

with eigenfunction x_1 and x_2

$$x_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{-t} \tag{10}$$

$$x_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{3t} \tag{11}$$

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