On solution of limit $\lim_{x \to 0}(1-2x)^{1/x}$

We try to find limit

$$\lim_{x \to 0}(1-2x)^{\frac{1}{x}} \tag{1}$$

We write

$$\lim_{x \to 0}(1-2x)^{\frac{1}{x}} = \lim_{x \to 0} e^{\ln(1-2x)^{\frac{1}{x}}} = e^{\lim_{x \to 0} \ln(1-2x)^{\frac{1}{x}}} = e^{\lim_{x \to 0} \frac{\ln(1-2x)}{x}} \tag{2}$$

We calculate the limit $\lim_{x \to 0} \frac{\ln(1-2x)}{x}$. We notice that $\lim_{x \to 0} \ln(1-2x) = \ln 1 = 0$ and that we may use the rule of l'Hôpital to calculate the limit of the last exponent expression of equation (2)

$$\lim_{x \to 0} \frac{\ln(1-2x)}{x} = \left|\frac{0}{0}\right| \overset{(\mathrm{H})}{=} \lim_{x \to 0} \frac{\frac{1}{1-2x}(-2)}{1} = \lim_{x \to 0} \frac{-2}{1-2x} = -2 \tag{3}$$

We arrive then at our result

$$\lim_{x \to 0}(1-2x)^{\frac{1}{x}} = e^{-2} \tag{4}$$