Note on division of polynomials

Example 1. Let us divide the polynomial $x^3 + x^2 - 10x + 8$ by $x - 5$

$$\begin{array}{rl} \big(\quad x^3 \; + x^2 - 10x \quad + 8 \big) \div \big(x - 5 \big) = x^2 + 6x + 20 + \dfrac{108}{x-5} \\ \underline{- x^3 + 5x^2} \\ \qquad\quad 6x^2 - 10x \\ \qquad \underline{- 6x^2 + 30x} \\ \qquad\qquad\quad 20x \quad + 8 \\ \qquad\qquad \underline{- 20x + 100} \\ \qquad\qquad\qquad\quad 108 \end{array} \tag{1}$$

The polynomial $x^3 + x^2 - 10x + 8$ is not divisible by $x - 5$. The number 108 is the remainder of the division.

Example 2. Let us divide the polynomial $2x^3 + 7x^2 + x - 10$ by $x - 1$

$$\begin{array}{rl} \big(\quad 2x^3 + 7x^2 \quad + x - 10 \big) \div \big(x - 1 \big) = 2x^2 + 9x + 10 \\ \underline{- 2x^3 + 2x^2} \\ \qquad\quad 9x^2 \quad + x \\ \qquad \underline{- 9x^2 \; + 9x} \\ \qquad\qquad\quad 10x - 10 \\ \qquad\qquad \underline{- 10x + 10} \\ \qquad\qquad\qquad\quad 0 \end{array} \tag{2}$$

The polynomial $2x^3 + 7x^2 + x - 10$ is divisible by $x - 1$ what means that the number 1 is a root of the polynomial $2x^3 + 7x^2 + x - 10$. This polynomial is also divisible by $2x + 5$ and by $x + 2$ what means that it also has the roots -2.5 and -2.

References

[1] Swietoslaw Romanowski and Wlodzimierz Wrona (1967) *Matematyka wyzsza dla studiow technicznych* Warszawa, Panstwowe Wydawnictwo Naukowe

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