

## Determinant 2 by 2

Let us consider the system of two linear equations

$$a_{1,1}x_1 + a_{1,2}x_2 = b_1 \quad (1)$$

$$a_{2,1}x_1 + a_{2,2}x_2 = b_2 \quad (2)$$

Let us multiply the equation (1) by  $a_{2,2}$  and the equation (2) by  $a_{1,2}$  what gives

$$a_{1,1}a_{2,2}x_1 + a_{1,2}a_{2,2}x_2 = b_1a_{2,2} \quad (3)$$

$$a_{1,2}a_{2,1}x_1 + a_{1,2}a_{2,2}x_2 = b_2a_{1,2} \quad (4)$$

Now we subtract equation (4) from equation (3) and we receive

$$(a_{1,1}a_{2,2} - a_{1,2}a_{2,1})x_1 = b_1a_{2,2} - b_2a_{1,2} \quad (5)$$

Let us multiply equation (1) by  $a_{2,1}$  and equation (2) by  $a_{1,1}$  to receive

$$a_{1,1}a_{2,1}x_1 + a_{1,2}a_{2,1}x_2 = b_1a_{2,1} \quad (6)$$

$$a_{1,1}a_{2,1}x_1 + a_{1,1}a_{2,2}x_2 = b_2a_{1,1} \quad (7)$$

and we subtract equation (6) from equation (7)

$$(a_{1,1}a_{2,2} - a_{1,2}a_{2,1})x_2 = b_2a_{1,1} - b_1a_{2,1} \quad (8)$$

Equations (5) and (8) can be rewritten as follows

$$\det A = a_{1,1}a_{2,2} - a_{1,2}a_{2,1} = \begin{vmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{vmatrix} \quad (9)$$

$$\det A_{x_1} = b_1a_{2,2} - b_2a_{1,2} = \begin{vmatrix} b_1 & a_{1,2} \\ b_2 & a_{2,2} \end{vmatrix} \quad (10)$$

$$\det A_{x_2} = b_2 a_{1,1} - b_1 a_{2,1} = \begin{vmatrix} a_{1,1} & b_1 \\ a_{2,1} & b_2 \end{vmatrix} \quad (11)$$

where  $\det A$  is determinant of the system of linear equations (1) and (2). To compute the  $x_1$  and  $x_2$  the determinants  $\det A_{x_1}$  and  $\det A_{x_2}$  are needed. Then we have

$$x_1 = \frac{\det A_{x_1}}{\det A} = \frac{\begin{vmatrix} b_1 & a_{1,2} \\ b_2 & a_{2,2} \end{vmatrix}}{\begin{vmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{vmatrix}} \quad (12)$$

and

$$x_2 = \frac{\det A_{x_2}}{\det A} = \frac{\begin{vmatrix} a_{1,1} & b_1 \\ a_{2,1} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{vmatrix}} \quad (13)$$

The way of computing the  $x_1$  and  $x_2$  using the equations (12) and (13) is called Cramer's rule. We see that if  $\det A = 0$  the system of linear equations (1) and (2) cannot be solved using Cramer's rule.

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