Note on Derivative of x^n

We attempt to derive a formula for the first derivative of x^n. We will use the formula for the derivative of the product of two functions, i.e.

$$(f(x)g(x))' = f'(x)g(x) + f(x)g'(x) \tag{1}$$

In the first step we rewrite x^n as a product of x and x^{n-1} and we compute the derivative of this product as follows:

$$\begin{aligned} (x^n)' &= (xx^{n-1})' \qquad (2) \\ &= x^{n-1} + x(x^{n-1})' \end{aligned}$$

In the second step we continue

$$\begin{aligned} & x^{n-1} + x(x^{n-1})' \qquad (3) \\ &= x^{n-1} + x(xx^{n-2})' \\ &= x^{n-1} + x(x^{n-2} + x(x^{n-2})') \\ &= x^{n-1} + x^{n-1} + x^2(x^{n-2})' \\ &= 2x^{n-1} + x^2(x^{n-2})' \end{aligned}$$

The third step is very similar

$$\begin{aligned} & 2x^{n-1} + x^2(x^{n-2})' \qquad (4) \\ &= 2x^{n-1} + x^2(xx^{n-3})' \\ &= 2x^{n-1} + x^2(x^{n-3} + x(x^{n-3})') \\ &= 2x^{n-1} + x^{n-1} + x^3(x^{n-3})' \qquad (5) \\ &= 3x^{n-1} + x^3(x^{n-3})' \end{aligned}$$

As a result in the n-th step we will receive

$$(x^n)' = nx^{n-1} + x^n(x^{n-n})'$$

Because

$$(x^{n-n})' = (x^0)' = (1)' = 0 \tag{6}$$

we receive

$$(x^n)' = nx^{n-1} \tag{7}$$