

Derivative of definite integral

Let us have the primary function of $f(x)$ as $F(x)$ and

$$\phi(a) = \int_0^a f(x) dx = F(a) - F(0) \quad (1)$$

We would like to compute the derivative $\frac{d\phi(a)}{da}$. We do it as follows

$$\frac{d\phi(a)}{da} = \frac{d}{da}(F(a) - F(0)) = \frac{dF(a)}{da} = f(a) \quad (2)$$

Let us look at an example. Let $f(x) = x^2$ and

$$\phi(a) = \int_0^a x^2 dx = \left[\frac{x^3}{3} \right]_0^a = \frac{a^3}{3} \quad (3)$$

$$\frac{d\phi(a)}{da} = \frac{d}{da} \frac{a^3}{3} = a^2 = f(a) \quad (4)$$

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