

Cycloid

Point P on the rim of a circle rolling on a straight line traces a curve named *cycloid*. Let us find the equation of this curve.

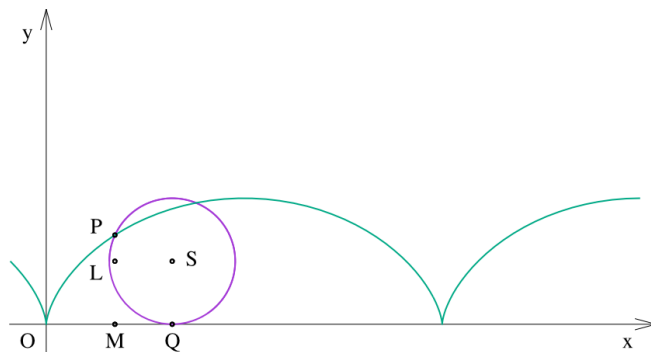


Figure 1: The point $P(x, y)$ on the rim of a circle rolling on the Ox axis in the positive direction of this axis traces a cycloid.

Let the circle roll on the Ox axis in the positive direction of this axis (Fig. 1). Let at the beginning the point P is located at the origin O of the system of coordinates. After some time the radius SP of the circle will have with the radius SQ , which is perpendicular to the Ox axis, an angle $\sphericalangle QSP = t$. The distance OQ is equal to the length of the arc PQ , and it is equal to rt where r is the radius length of the rolling circle. We notice that $LS = MQ$ and $LM = SQ$. For the coordinates of the point $P(x, y)$ we obtain the formulas

$$x = OQ - MQ = rt - r \cos(t - \pi/2)$$

$$y = ML + LP = r + r \sin(t - \pi/2)$$

The parametric equations of the cycloid are then

$$x = r(t - \sin(t)) \tag{1}$$

$$y = r(1 - \cos(t)) \tag{2}$$

Cycloid is a periodic curve of a period equal to 2π . The cycloid for a circle of radius one is presented in figure 1.

References

- [1] Swietoslaw Romanowski and Włodzimierz Wrona (1967) *Matematyka wyższa dla studiów technicznych* Warszawa, Państwowe Wydawnictwo Naukowe

Paweł Jan Piskorz (paweljs@gmail.com)