

arctan(x) derivative

We have

$$y = \tan x$$

and

$$x = \arctan y$$

$$\frac{dy}{dx} = \frac{d \tan x}{dx} = \frac{d}{dx} \left(\frac{\sin x}{\cos x} \right) = \frac{\sin^2 x + \cos^2 x}{\cos^2 x} = \frac{\tan^2 x + 1}{1}$$

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Therefrom

$$\frac{dy}{dx} = \frac{y^2 + 1}{1}$$

and

$$\frac{dx}{dy} = \frac{d \arctan y}{dy} = \frac{1}{y^2 + 1}$$

what can be rewritten as

$$\frac{d \arctan x}{dx} = \frac{1}{x^2 + 1}$$

arctan(x) derivative

We have then

$$\int \frac{1}{x^2 + 1} dx = \arctan x + C$$