

Simple fractions decomposition examples

Example 1. Let us decompose into simple fractions

$$\frac{2}{x^2 - 1} \tag{1}$$

Solution:

$$\frac{2}{x^2 - 1} = \frac{2}{(x - 1)(x + 1)} = \frac{A^{(1)}}{x - 1} + \frac{A^{(2)}}{x + 1} \tag{2}$$

Let us multiply equation (2) by $x - 1$ and compute the limits

$$\begin{aligned} & \lim_{x \rightarrow 1} \frac{2}{(x - 1)(x + 1)}(x - 1) = \tag{3} \\ & = \lim_{x \rightarrow 1} \frac{A^{(1)}}{x - 1}(x - 1) + \lim_{x \rightarrow 1} \frac{A^{(2)}}{x + 1}(x - 1) = \frac{2}{1 + 1} = A^{(1)} = 1 \end{aligned}$$

$$\begin{aligned} & \lim_{x \rightarrow -1} \frac{2}{(x - 1)(x + 1)}(x + 1) = \tag{4} \\ & = \lim_{x \rightarrow -1} \frac{A^{(1)}}{x - 1}(x + 1) + \lim_{x \rightarrow -1} \frac{A^{(2)}}{x + 1}(x + 1) = \frac{2}{-1 - 1} = A^{(2)} = -1 \end{aligned}$$

Then we have the decomposition

$$\frac{2}{x^2 - 1} = \frac{2}{(x - 1)(x + 1)} = \frac{1}{x - 1} - \frac{1}{x + 1} \tag{5}$$

References

- [1] Saff, E.B., Snider, A.D. (2014) *Fundamentals of Complex Analysis With Applications to Engineering, Science, and Mathematics* Pearson

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