Radioactive Decay and Poisson Distribution

Let us consider certain amount of the radioactive substance, of a very long half-life, which is sending α particles. The probability that k α particles will reach in a certain period of time a given part of space is given by the formula

$$p(k, \lambda) = \frac{\lambda^k}{k!} e^{-\lambda} \tag{1}$$

To determine the number λ let us perform a large number N of measurements of number of α particles during certain period of time reaching a given part of space. Let N_k denotes the number of measurements in which exactly k α particles occurred. We have then

$$N_0 + N_1 + N_2 + \ldots = N \tag{2}$$

In addition, we expect that $N_k \approx N p(k, \lambda)$.

The total number M of α particles noticed in N measurements is therefore

$$\begin{aligned} M &= N_1 + 2N_2 + 3N_3 + \ldots = \\ &= N \cdot p(1, \lambda) + 2N \cdot p(2, \lambda) + 3N \cdot p(3, \lambda) + \ldots = \\ &= N \frac{\lambda^1}{1!} e^{-\lambda} + 2N \frac{\lambda^2}{2!} e^{-\lambda} + 3N \frac{\lambda^3}{3!} e^{-\lambda} + \ldots = \\ &= N e^{-\lambda} \lambda \left(1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \ldots \right) = N e^{-\lambda} \lambda e^{\lambda} \approx N\lambda \end{aligned} \tag{3}$$

Therefrom we have that

$$\lambda = \frac{M}{N} \tag{4}$$

References

[1] Wlodzimierz Wrona (1969) *Matematyka* Czesc I, Warszawa, Panstwowe Wydawnictwo Naukowe

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