Notes for Poisson Process

We say that random variable η has the exponential distribution of rate $\lambda > 0$ if

$$P\{\eta > t\} = e^{-\lambda t} \tag{1}$$

for all $t > 0$.

For example, the emission of particles by a sample of radioactive material or calls made at a telephone exchange, occur at random times. The probability that no particle is emitted (no call is made) up to time t is known to decay exponentially as t increases. That is to say, the time η of the first emission has the exponential distribution

$$P\{\eta > t\} = e^{-\lambda t} \tag{2}$$

Notes for Poisson Process Probabilities

Messages arrive at the switch board in a Poisson manner at an average of six per hour. Find the probability of each of the following events:

1. Exactly two messages arrive in one hour
2. No messages arrive in one hour
3. At least three messages arrive within one hour

Solution:

Let K be the random variable that denotes the number of messages arriving at the switch board within one hour interval. The probability that random variable K assumes the value k is given by

$$p_K(k) = \frac{6^k}{k!}e^{-6} \tag{3}$$

for $k = 0, 1, 2, \ldots$.

1. The probability that exactly two messages arrive within one hour is

$$p_K(2) = \frac{6^2}{2!}e^{-6} = 18e^{-6} = 0.0446 \tag{4}$$

2. The probability that no message arrives within one hour is

$$p_K(0) = \frac{6^0}{0!}e^{-6} = e^{-6} = 0.0025 \tag{5}$$

3. The probability that at least three messages arrive within one hour is

$$\begin{aligned} p_K(k >= 3) = 1 - p_K(k < 3) &= 1 - \{p_K(0) + p_K(1) + p_K(2)\} \\ = 1 - e^{-6}\{\frac{6^0}{0!} + \frac{6^1}{1!} + \frac{6^2}{2!}\} &= 1 - e^{-6}\{1 + 6 + 18\} \\ &= 1 - 25e^{-6} = 0.9380 \end{aligned} \tag{6}$$

Note on Probablity of No Event in the Poisson Process

The probability that no message arrives within one hour is

$$p_K(0) = \frac{6^0}{0!}e^{-6} = e^{-6} = 0.0025 \tag{7}$$

The probability that no message arrives within next hour is also

$$p_K(0) = \frac{6^0}{0!}e^{-6} = e^{-6} = 0.0025 \tag{8}$$

so the probability that no message arrives within two hours is

$$p_K(0)p_K(0) = e^{-6}e^{-6} = e^{-6\cdot 2} \tag{9}$$

Probability that no message arrives within t hours is

$$e^{-6t} \tag{10}$$

and in general for the rate of message arrival λ the probability that no message arrives within t hours is

$$e^{-\lambda t} \tag{11}$$