Note on straight line equation

We want to determine the equation of a straight line going through two different points (x_1, y_1) and (x_2, y_2).

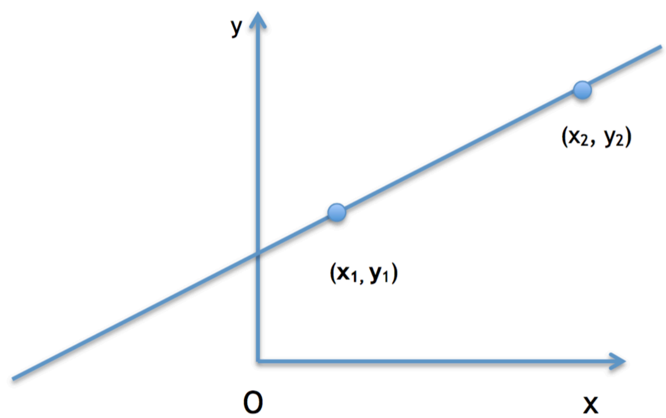

Figure 1: The straight line has two points of coordinates (x_1, y_1) and (x_2, y_2), or in other words the straight line is going through two points (x_1, y_1) and (x_2, y_2).

We have the following ratios equal

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1} \tag{1}$$

for any values of x, y. We can write

$$(y - y_1)(x_2 - x_1) = (y_2 - y_1)(x - x_1) \tag{2}$$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \tag{3}$$

$$y = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) + y_1 \tag{4}$$

$$y = \frac{y_2 - y_1}{x_2 - x_1}x + \frac{y_2 - y_1}{x_2 - x_1}(-x_1) + y_1\frac{x_2 - x_1}{x_2 - x_1} \tag{5}$$

$$y = \frac{y_2 - y_1}{x_2 - x_1}x + \frac{-x_1y_2 + x_1y_1 + x_2y_1 - x_1y_1}{x_2 - x_1} \tag{6}$$

$$y = \frac{y_2 - y_1}{x_2 - x_1}x + \frac{x_2y_1 - x_1y_2}{x_2 - x_1} \tag{7}$$

The equation (7) is an equation of a straight line going through two points of coordinates (x_1, y_1) and (x_2, y_2). We see that this equation has the form

$$y = ax + b \tag{8}$$

where

$$a = \frac{y_2 - y_1}{x_2 - x_1} \tag{9}$$

and

$$b = \frac{x_2y_1 - x_1y_2}{x_2 - x_1} \tag{10}$$

a is the tangent of the angle the straight line has with the Ox axis and b is one of coordinates of the point $(0, b)$ which is the point of intersection of the straight line $y = ax + b$ with the axis Oy.

Pawel Jan Piskorz (paweljs@gmail.com)