

# Michaelis-Menten kinetics of enzymatic reactions



We omit in the above equation the reaction which is reverse to the decomposition of the complex  $(ES)^\ddagger$  into product. The appropriate kinetic equations have the form

$$-\frac{d[S]}{dt} = k_2[(ES)^\ddagger] \quad (2)$$

$$\frac{d[(ES)^\ddagger]}{dt} = k_1[E][S] - k_{-1}[(ES)^\ddagger] - k_2[(ES)^\ddagger] \quad (3)$$

We use the steady state method ( $[(ES)^\ddagger] = const$ ) obtaining

$$\frac{d[(ES)^\ddagger]}{dt} = 0 \quad (4)$$

and

$$[(ES)^\ddagger] = \frac{k_1[E][S]}{k_{-1} + k_2} \quad (5)$$

If we assume that the initial concentrations of  $S$  and  $E$  fulfill the condition that  $[S]_0 \gg [E]_0$  and we take into account that that

$$[E]_0 = [E] + [(ES)^\ddagger] \quad (6)$$

we may insert (6) into (5) and we obtain

$$[(ES)^\ddagger](k_{-1} + k_2) = k_1[E][S] \quad (7)$$

$$[E] = [E]_0 - [(ES)^\ddagger] \quad (8)$$

$$[(ES)^\ddagger](k_{-1} + k_2) = k_1([E]_0 - [(ES)^\ddagger])[S] \quad (9)$$

$$[(ES)^\ddagger](k_{-1} + k_2 + k_1[S]) = k_1[E]_0[S] \quad (10)$$

$$[(ES)^\ddagger] = \frac{k_1[E]_0[S]}{k_{-1} + k_2 + k_1[S]} \quad (11)$$

Now we can write using (2)

$$-\frac{d[S]}{dt} = \frac{k_1 k_2 [E]_0 [S]}{k_{-1} + k_2 + k_1 [S]} \quad (12)$$

Now we divide the numerator and denominator in equation (12) by  $k_1$  obtaining

$$-\frac{d[S]}{dt} = \frac{k_2 [E]_0 [S]}{(k_{-1} + k_2)/k_1 + [S]} = \frac{k_2 [E]_0 [S]}{k_M + [S]} \quad (13)$$

where

$$\frac{k_{-1} + k_2}{k_1} = k_M \quad (14)$$

and the  $k_M$  is called the Michaelis-Menten constant.

$$v = -\frac{d[S]}{dt} \quad (15)$$

$v$  is maximal if  $[S] \rightarrow \infty$

$$v_{max} = \lim_{[S] \rightarrow \infty} \frac{k_2 [E]_0 [S]}{k_M + [S]} = \lim_{[S] \rightarrow \infty} \frac{k_2 [E]_0}{\frac{k_M}{[S]} + 1} = k_2 [E]_0 \quad (16)$$

and the speed of the reaction is proportional to the concentration of the enzyme. If the concentration of the substrate  $[S]$  is small then

$$-\frac{d[S]}{dt} = \lim_{[S] \rightarrow 0} \frac{k_2 [E]_0 [S]}{k_M + [S]} = \frac{k_2}{k_M} [E]_0 [S] \quad (17)$$

and we have the reaction of the first order. The speed of the reaction expressed in the equation (16) is maximal for a given concentration of the enzyme  $[E]_0$ . Substituting  $v_{max}$  from the equation (16) into the equation (13) we obtain

$$v = \frac{v_{max} [S]}{k_M + [S]} \quad (18)$$

Taking the inverse of equation (18) we obtain

$$\frac{1}{v} = \frac{k_M}{v_{max}[S]} + \frac{1}{v_{max}} \quad (19)$$

We may plot  $1/v$  as the function of  $1/[S]$  and we obtain a straight line, the slope of which is  $k_M/v_{max}$ , and the point  $(0, 1/v_{max})$  is the point where the plot has a common point with the  $1/v$  axis.

## References

- [1] Sobczyk, L., Kisza, A. (1975) *Chemia fizyczna dla przyrodników* Państwowe Wydawnictwo Naukowe, Warszawa

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