Solving the integral $\int_0^\infty e^{-ax}\cos(\mu x)\,dx$

$$\int e^{-ax}\cos(\mu x)\,dx = \left|\begin{array}{ccc} \mu x & = & v \\ dx & = & \frac{1}{\mu}dv \\ x & = & \frac{v}{\mu} \end{array}\right| = \frac{1}{\mu}\int e^{-\frac{a}{\mu}v}\cos(v)\,dv \tag{1}$$
$$= \frac{1}{\mu}\frac{e^{-\frac{a}{\mu}v}}{1+\frac{a^2}{\mu^2}}\Big(\sin(v) - \frac{a}{\mu}\cos(v)\Big) = L(v)$$

$$L(v=\infty) = 0 \tag{2}$$
$$L(v=0) = -\frac{a}{\mu^2+a^2} \tag{3}$$

$$\int_0^\infty e^{-ax}\cos(\mu x)\,dx = L(v=\infty) - L(v=0) = \frac{a}{\mu^2+a^2} \tag{4}$$