Integration of $\sqrt{x^2+1}$

In

$$\int \sqrt{x^2+1}\,dx$$

let's substitute $x = \tan\theta$

$$\sqrt{x^2+1} = \sqrt{\tan^2\theta+1} = \sqrt{\frac{\sin^2\theta}{\cos^2\theta}+\frac{\cos^2\theta}{\cos^2\theta}} = \sec\theta$$

$$dx = \sec^2\theta\,d\theta$$

$$\int \sqrt{x^2+1}\,dx = \int \sec^3\theta\,d\theta$$

Integration of $\sqrt{x^2+1}$

$$\int \sqrt{x^2+1}\,dx = \int \sec^3\theta\,d\theta$$

$$= \frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln\left|\sec\theta+\tan\theta\right| + C$$

$$= \frac{1}{2}x\sqrt{x^2+1} + \frac{1}{2}\ln\left|x+\sqrt{x^2+1}\right| + C$$