Integration of $\sqrt{x^2-1}$

In

$$\int \sqrt{x^2-1}\,dx$$

let's substitute $x = \sec\theta$

$$\sqrt{x^2-1} = \sqrt{\sec^2\theta - 1} = \sqrt{\tan^2\theta} = \tan\theta$$

$$dx = \sec\theta\tan\theta\,d\theta$$

$$\int \sqrt{x^2-1}\,dx = \int \sec\theta\tan^2\theta\,d\theta$$

Integration of $\sqrt{x^2-1}$

$$\int \sqrt{x^2-1}\,dx = \int \sec\theta \tan^2\theta\,d\theta$$

$$= \int \sec^3\theta \sin^2\theta\,d\theta = \int \sec^3\theta(1-\cos^2\theta)\,d\theta$$

$$= \int \sec^3\theta(1-\sec^{-2}\theta)\,d\theta = \int \sec^3\theta\,d\theta - \int \sec\theta\,d\theta$$

$$= \frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\int \sec\theta\,d\theta - \int \sec\theta\,d\theta$$

Integration of $\sqrt{x^2-1}$

$$\int \sqrt{x^2-1}\,dx = \frac{1}{2}\sec\theta\tan\theta - \frac{1}{2}\int \sec\theta\,d\theta$$

$$= \frac{1}{2}\sec\theta\tan\theta - \frac{1}{2}\ln|\sec\theta+\tan\theta| + C$$

$$= \left| \begin{array}{lcl} \tan\theta & = & \sqrt{x^2-1} \\ \sec\theta & = & x \end{array} \right| =$$

$$= \frac{1}{2}x\sqrt{x^2-1} - \frac{1}{2}\ln\left|x+\sqrt{x^2-1}\right| + C$$