Integration of $\sqrt{1-x^2}$

In

$$\int \sqrt{1-x^2}\,dx$$

let's substitute $x = \sin\theta$

$$\sqrt{1-x^2} = \sqrt{1-\sin^2\theta} = \sqrt{\cos^2\theta} = \cos\theta$$

$$dx = \cos\theta\,d\theta$$

$$\int \sqrt{1-x^2}\,dx = \int \cos^2\theta\,d\theta$$

Integration of $\sqrt{1-x^2}$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = \cos^2\theta - (1-\cos^2\theta)$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$\cos^2\theta = \frac{\cos 2\theta + 1}{2}$$

$$\int \cos 2\theta \, d\theta = \left| \begin{array}{lcl} 2\theta & = & u \\ 2d\theta & = & du \end{array} \right| = \frac{1}{2}\int \cos u \, du =$$

$$= \frac{1}{2}\sin u + C_1$$

Integration of $\sqrt{1-x^2}$

$$\int \cos 2\theta \, d\theta = \frac{1}{2}\sin 2\theta + C_1 = \sin\theta\cos\theta + C_1$$

$$\int \cos^2\theta \, d\theta = \int \frac{\cos 2\theta + 1}{2} \, d\theta$$

$$\int \cos^2\theta \, d\theta = \frac{1}{2}\theta + \frac{1}{2}\sin\theta\cos\theta + C$$

$$\int \sqrt{1-x^2} \, dx = \frac{1}{2}\arcsin x + \frac{1}{2}x\sqrt{1-x^2} + C$$