Solving the Integral $\int dx/(x^2+x+1)$

$$x^2+x+1 = \left[\left(x+\frac{1}{2}\right)^2 + 1 - \frac{1}{4}\right] = \left(x+\frac{1}{2}\right)^2 + \frac{3}{4} \tag{1}$$

$$\begin{aligned}\int \frac{dx}{x^2+x+1} &= \int \frac{dx}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}} = \left|\begin{array}{rcl} x+\frac{1}{2} &=& u \\ dx &=& du \end{array}\right| = \int \frac{du}{u^2+\frac{3}{4}} = \\ &= \int \frac{du}{\frac{3}{4}\left(\frac{4}{3}u^2+1\right)} = \left|\begin{array}{rcl} \frac{4}{3}u^2 &=& \left(\frac{2}{\sqrt{3}}u\right)^2 \\ w &=& \frac{2}{\sqrt{3}}u \\ du &=& \frac{\sqrt{3}}{2}dw \end{array}\right| = \frac{4}{3}\frac{\sqrt{3}}{2}\int \frac{dw}{w^2+1} = \\ &= \frac{2}{\sqrt{3}}\arctan(w) + C = \frac{2}{\sqrt{3}}\arctan\left(\frac{2}{\sqrt{3}}u\right) + C = \\ &= \frac{2}{\sqrt{3}}\arctan\left(\frac{2}{\sqrt{3}}\left(x+\frac{1}{2}\right)\right) + C\end{aligned} \tag{2}$$