On derivation of the Fick's law

Let us consider movement of the particles due to diffusion along the Ox axis. Half of the particles $N(x,t)$ at point x moves to the right and half of the particles $N(x+\Delta x,t)$ at point $x+\Delta x$ moves to the left. The increase of the number of particles in the area between x and $x+\Delta x$ is therefore

$$\frac{1}{2}N(x,t)-\frac{1}{2}N(x+\Delta x,t)=-\frac{1}{2}\Big(N(x+\Delta x,t)-N(x,t)\Big)=-\frac{1}{2}\Delta N(x,t) \quad (1)$$

The flux causing the increase of the number of particles in the volume between x and $x+\Delta x$ with the cross section area a per time Δt is

$$F=-\frac{1}{2}\left[\frac{N(x+\Delta x,t)}{a\Delta t}-\frac{N(x,t)}{a\Delta t}\right]=-\frac{1}{2}\left[\frac{\Delta N(x,t)}{a\Delta t}\right] \quad (2)$$

where a is the cross section area of the system of particles. We have then

$$\begin{aligned} F &= -\frac{1}{2}\frac{\Delta x}{\Delta t}\left[\frac{\Delta N(x,t)}{a\Delta x}\right] \\ &= -\frac{1}{2}\frac{\Delta x}{\Delta t}\Delta c(x,t)=-\frac{1}{2}\frac{(\Delta x)^2}{\Delta t}\left[\frac{\Delta c(x,t)}{\Delta x}\right] \end{aligned} \quad (3)$$

where c is concentration. At $\Delta x\to 0$ and $\Delta t\to 0$ we have

$$F=-D\frac{\partial c}{\partial x} \quad (4)$$

where D is the diffusion coefficient. Equation (4) expresses the Fick's law in one dimension. In general in three dimensions for the flux $\vec{F}$ we receive the equation for the Fick's law

$$\vec{F}=-D\,\nabla C \quad (5)$$

The Fick's law states that the flux of particles at a given point of the system where the concentration of particles is $C(x,t)$ is proportional to the gradient of the particles concentration and it occurs in the direction which is opposite to the gradient of the concentration. From equation (3) we see that the diffusion coefficient

$$D=\frac{1}{2}\frac{(\Delta x)^2}{\Delta t} \quad (6)$$

has a meaning of the halved squared distance the particle moves due to diffusion per unit time.

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