Bernoulli inequality

When $A > 0$ then for all natural numbers $n \geq 2$ occurs the Bernoulli inequality

$$(1 + A)^n > 1 + nA \tag{1}$$

To proove the Bernoulli inequality we use mathematical induction.

Proof. We start checking the inequality for $n = 2$. We have then

$$(1 + A)^2 = 1 + 2A + A^2 \tag{2}$$

Because $A^2 > 0$ it occurs

$$(1 + A)^2 > 1 + 2A \tag{3}$$

and the Bernoulli inequality is proven for $n = 2$.

Next we proove that if the inequality occurs for $n = k$ it also occurs for $n = k + 1$. Assuming that

$$(1 + A)^k > 1 + kA \tag{4}$$

we want to show that

$$(1 + A)^{k+1} > 1 + (k + 1)A \tag{5}$$

Let us take into account that

$$(1 + A)^{k+1} = (1 + A)^k(1 + A) > (1 + kA)(1 + A) = 1 + A + kA + kA^2 \tag{6}$$

Because $kA^2 > 0$ we receive

$$(1 + A)^{k+1} > 1 + A(k + 1) \tag{7}$$

or

$$(1 + A)^{k+1} > 1 + (k + 1)A \tag{8}$$

According to the mathematical induction the Bernoulli inequality is valid for all natural numbers $n \geq 2$.

References

[1] Swietoslaw Romanowski and Wlodzimierz Wrona (1967) *Matematyka wyzsza dla studiow technicznych* Warszawa, Panstwowe Wydawnictwo Naukowe

Pawel Jan Piskorz (paweljs@gmail.com)