

What is $9.999\dots$ equal to

We have

$$\begin{aligned} 9.999\dots &= 9 * 1 + 9 * \left(\frac{1}{10}\right) + 9 * \left(\frac{1}{100}\right) + 9 * \left(\frac{1}{1000}\right) + \dots & (1) \\ &= 9 * \left(1 + \frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3} + \dots\right) \\ &= 9 * \frac{1}{1 - \frac{1}{10}} = 9 * \frac{1}{\frac{9}{10}} = 10 \end{aligned}$$

We used here the formula for the sum of a geometric series

$$\frac{1}{1 - x} = 1 + x + x^2 + x^3 + \dots \quad (2)$$

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